puter Program," TN HSA 157, Oct. 1969, Weapons Research Establishment, Salisbury, South Australia.

[7] Gilbert, N. E., "Free Flight Measurement of Aerodynamic Lateral Force and Moment Coefficients on Bombs with Freely Spinning Cruciform and Monoplane Tails and Fixed Split Skirts," TN HSA 162, Nov. 1969, Weapons Research Establishment, Salisbury, South Australia.

[8] Gilbert, N. E., "The Use of Rate Gyroscopes in the Free Flight Measurement of Aerodynamic Lateral Force and Moment Coefficients," TN HSA 164, March 1970, Weapons Research Establishment, Salisbury, South Australia.

[9] Kolk, W. R., *Modern Flight Dynamics*, Space Technology Series, Prentice-Hall, Englewood Cliffs, N. J., 1961, pp. 6–11.

[10] Regan, F. J., Falusi, M. E., and Holmes, J. E., "Static Wind Tunnel Tests of the M823 Research Store with Fixed and Free-Spinning Tails," NOLTR 65-14, April 1967, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Md.

[11] Regan, F. J., Holmes, J. E., and Falusi, M. E., "Magnus Measurements on the M823 Research Store with Fixed and Freely Spinning Cruciform Stabilizers, Freely Spinning Monoplane Stabilizers and Split-Skirt Stabilizers," NOLTR 69-214, Nov. 1969, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Md.

Minimum Range-Sensitivity Deorbit

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Nomenclature

A = reference area for entry vehicle aerodynamic coefficients
C_D = entry vehicle drag coefficient
e = orbit eccentricity
h = altitude
L = angular momentum per unit mass
r = central radius
V = velocity
ΔV = retrovelocity impulse
W = entry vehicle weight
β = retrorocket alignment angle
γ = path angle (down from local horizontal)
η = true anomaly
θ = range angle from deorbit to entry
μ = planetary gravitational constant

Subscripts

o = initial unperturbed orbit condition
1 = condition immediately subsequent to retrofire
E = condition at atmospheric entry

Introduction

SINCE a principal contributer to the impact point dispersion of a deorbiting ballistic atmospheric entry vehicle is often retrorocket alignment error, the fact that a minimum range, minimum range-sensitivity alignment exists has long been of interest to designers of this type of spacecraft. Approximate equations describing this maneuver were published by Low[1] some years ago, showing that for a given retrovelocity ΔV, range dispersion is minimized (neglecting postentry atmospheric effects) when the retro-

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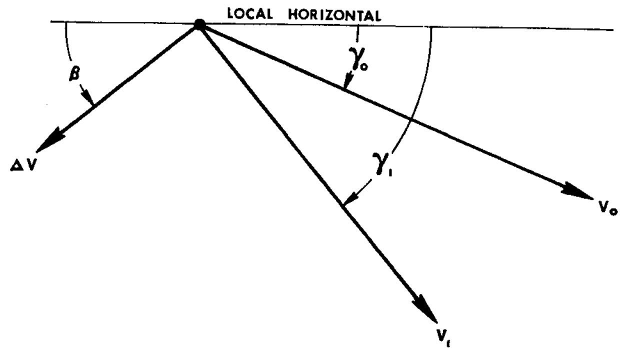

Fig. 1 Notation at deorbit point.

velocity vector is so aligned that $\partial\theta/\partial\beta = 0$. This maneuver may be contrasted to the minimum energy deorbit, of utility for lifting entry vehicles to minimize propulsion weight, where it is desired to achieve the steepest entry path angle possible for a given ΔV. An exact expression for the latter case has been developed previously,[2] and here a similar equation is presented for the minimum range-sensitivity deorbit which is also exact within the assumptions made and equally valid for elliptical orbits.

Analysis

In an inverse squared, drag-free force field, the basic conic orbit equation may be written in general as

$$1/r = 1 + e \cos\eta/(L^2/\mu) \tag{1}$$

Then the central radius at the atmospheric entry condition may be related to that immediately subsequent to retrofire by

$$1/r_E - 1/r_1 = (\mu e/L^2)(\cos\eta_E - \cos\eta_1) \tag{2}$$

Defining the range angle $\theta \equiv \eta_E - \eta_1$, Eq. (2) can be rewritten (following Cain[3]) by using the appropriate trigonometric formulas

$$1/r_E - 1/r_1 = -(\mu e/L^2)[\sin\eta_1 \sin\theta + \cos\eta_1(1 - \cos\theta)] \tag{3}$$

Now it is desirable to eliminate η_1 from the expression. Immediately from Eq. (1)

$$\cos\eta_1 = 1/e[(L^2/\mu r_1) - 1] \tag{4}$$

Furthermore, the path angle γ_1 may be expressed for a conic, and considering Fig. 1

$$\tan\gamma_1 = \frac{-e \sin\eta_1}{1 + e \cos\eta_1} = \frac{V_0 \sin\gamma_0 + \Delta V \sin\beta}{V_0 \cos\gamma_0 - \Delta V \cos\beta} \tag{5}$$

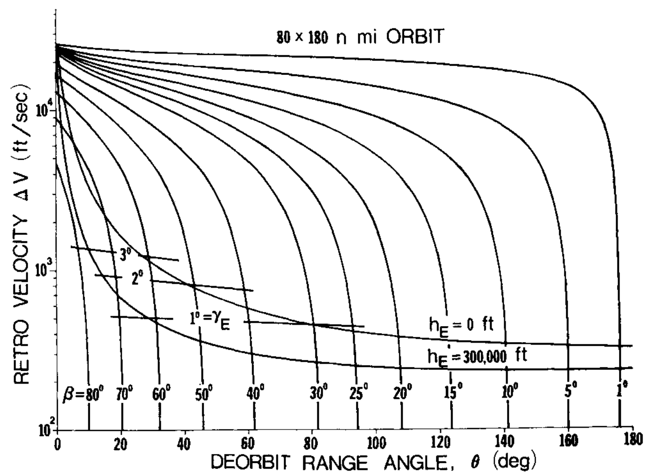

Fig. 2 Deorbit range angle vs ΔV, perigee deorbit.

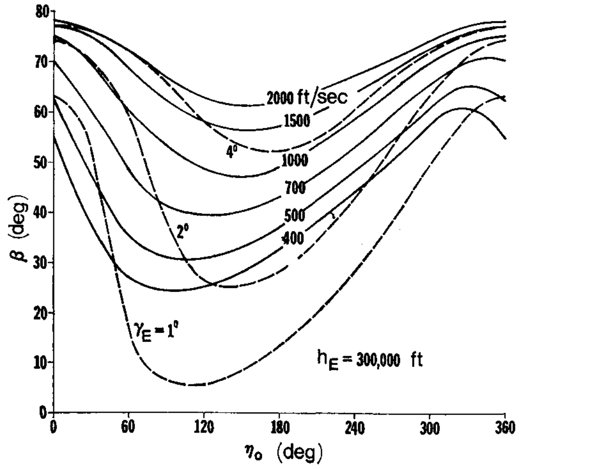

Fig. 3 Effect of deorbit location on β with ΔV or entry path angle constant.

The angular momentum per unit mass may be expressed

$$L = L_1 = L_E = r_o V_1 \cos\gamma_1 = r_o(V_o \cos\gamma_o - \Delta V \cos\beta) \quad (6)$$

Utilizing Eqs. (1) and (6) in (5)

$$\sin\eta_1 = -(L/\mu e)(V_o \sin\gamma_o + \Delta V \sin\beta) \quad (7)$$

Then substituting Eqs. (4) and (7) in (3)

$$1/r_E - 1/r_o = (V_o \sin\gamma_o + \Delta V \sin\beta)\sin\theta/L + [(\mu/L^2) - (1/r_o)](1 - \cos\theta) \quad (8)$$

Note that Eq. (8) may be expressed functionally as

$$\theta = \theta[\beta,\Delta V,L(\beta,\Delta V)] = \theta(\beta,\Delta V)$$

and the desired constraint to be imposed reflects the condition whereby the range is insensitive to retrorocket alignment. Then, differentiating Eq. (8) with ΔV held constant

$$\left.\frac{\partial\theta}{\partial\beta}\right|_{\Delta V} = 0 = -\frac{2\mu}{L^2}\frac{1-\cos\theta}{\sin\theta}\frac{\partial L}{\partial\beta} - \frac{V_0\sin\gamma_0 + \Delta V\sin\beta}{L}\frac{\partial L}{\partial\beta} + \Delta V\cos\beta \quad (9)$$

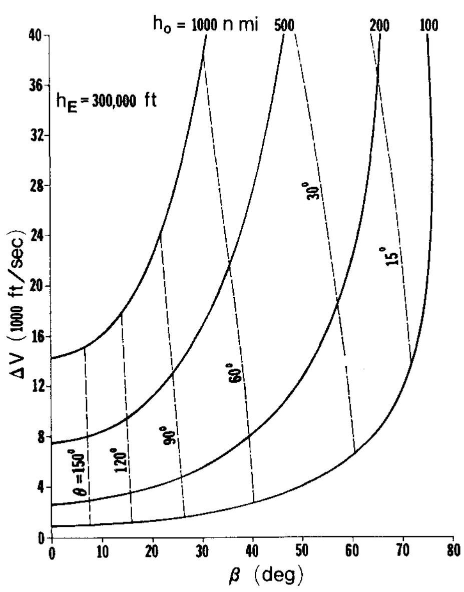

Fig. 4 Retrovelocity vs β, circular orbits.

Utilizing Eq. (6)

$$(\partial L/\partial\beta) = r_0 \Delta V \sin\beta \quad (10)$$

and substituting this into Eq. (9)

$$\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2} = \frac{L^2}{2\mu}\left[\frac{\cos\beta}{r_o\sin\beta} - \frac{V_o\sin\gamma_o + \Delta V\sin\beta}{L}\right] \quad (11)$$

or, with further manipulation

$$\theta = 2\arctan\{(L/2\mu\sin\beta)[V_o\cos(\gamma_o+\beta) - \Delta V]\} \quad (12)$$

which is the desired minimum range-sensitivity constraint. Parametric solution of Eq. (12) establishes a minimum range angle for each combination of β, ΔV selected. The corresponding entry radius (and hence altitude) can be obtained from Eq. (7), while entry velocity and path angle can readily be calculated from standard conic equations which need not be given here. Thus a particular set of entry conditions (but not necessarily corresponding to a desired entry altitude) is obtained for each β, ΔV set. With the basic approach employed here, other formulations could be obtained with resulting equations of second degree in ΔV or fourth degree in β. Neither of these appears to be as tractable as the present solution of first degree in θ.

Results

Typical parametric results obtained with the aid of a digital computer are presented for a 80 × 180 naut mile Earth orbit and for circular orbits from 100 to 1000 naut mile in altitude. One form the resulting data can take is illustrated by Fig. 2, which represents perigee deorbit for the elliptical case. Constant entry altitude and path angle lines have been imposed. Note the sensitivity of β to the choice of entry altitude. This suggests that for practical calculations involving atmospheric entry, care must be taken in selecting an "effective" entry altitude which reflects due consideration of entry vehicle W/C_DA such that range to impact is effectively minimized. When computer results are interpolated for a desired entry altitude, more convenient data presentations for the specified orbit can be constructed, as illustrated by Fig. 3 which gives the optimum β for given retrovelocity around the orbit and for given entry path angle. It is apparent that the optimum alignment is quite sensitive to the initial true anomaly. Figure 4 shows similar data for a family of circular earth orbits. Note that the optimum β is also very sensitive to the orbital altitude.

Concluding Remarks

An exact expression has been derived for deorbit from elliptical orbits with minimum range-sensitivity. While this result is limited to the orbital plane case, extension to out-of-plane deorbit maneuvers should be straightforward. Some effort was applied to reduce the present result to the approximate form obtained by Low[1] for near-Earth circular orbits; however, this was not successful due to differences in variables employed. Direct comparison between minimum energy and minimum range-sensitivity retrovelocity, say for a given entry path angle, which might be a tradeoff of interest for semiballistic entry vehicles having some limited atmospheric maneuvering capability, is possible if not straightforward using the present expression in conjunction with the corresponding principal equation of Ref. 2.

References

[1] Low, G. M., "Nearly Circular Transfer Trajectories for Descending Satellites," TR R-3, 1959, NASA.

[2] Galman, B. A., "Minimum Energy Deorbit," *Journal of Spacecraft and Rockets*, Vol. 3, No. 7, July 1966, pp 1030–1033; also "Erratum: Minimum Energy Deorbit," *Journal of Spacecraft and Rockets*, Vol. 3, No. 11, Nov. 1966, p 1696.

[3] Cain, B. J., "Minimum Range Deorbit with Application to Minimum Dispersion," TIS 66SD283, July 8, 1966, General Electric Co., Philadelphia, Pa.